IMTC 2003 – Instrumentation and Measurement
Technology Conference
Vail, CO, USA, 20-22 May 2003

Spectra Analysis of Nonuniformly Holding Signals for Time-Interleaved Systems with Timing Mismatches

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Abstract – This paper will present a complete analysis of signal spectra for general time-interleaved systems with the practical nonuniformly holding outputs imposed by the timing-mismatch effects. The analysis reveals first the spectra representation for different-form nonuniformly holding signal and then their closed-form expressions of the signal-to-noise-ratio (SNR), in terms of the number of channels, signal frequency, and jitter errors. Such analysis describes both the timing errors imposed by random clock-jitter and fixed periodic clock-skew. MATLAB simulation results are then presented to illustrate the effectiveness of the derived formula.

I. INTRODUCTION

The rapid evolution of electronic instruments and data communication demands high-speed data acquisition and conversion channels as well as signal processing units. Time-interleaved (TI) architecture is one of the most effective ways to boost the maximum speed of the analog electronics devices in current process technology e.g. time-interleaved ADC, DAC [1-2] and sampled-data filters [3-5]. However, the timing-mismatch of clock phases among different TI paths can greatly degrade the system performance [6-9], especially for high-speed operation. Such timing-mismatches have generally two different aspects: the first is caused by the random clock jitter which results in an increased noise floor over all frequencies, and the second one is due to the unmatched but fixed periodic clock timing-skew/offset among different channels that leads to the appearance of image sidebands in the frequency locations of the multiples of f_s/M (f_s – sampling frequency, M – period of timing-skew).

If the **I**nput signal is sampled by the system with **N**onuniform time-interval and later played out or equivalently represented by discrete samples in the **O**utput at **U**niform time instants for latter process, it can be designated by an **IN-OU** process, which is typical in the analog to digital conversion path (timing-mismatch only @ input signal sampling), e.g. TI ADCs [1] or multirate sampled-data decimators [4], as shown in Fig.1(a) with the correspondent signal waveform. While if the **I**nput signal is sampled by the system with **U**niformly spaced time-interval and later the samples are played out in the **O**utput **N**onuniformly, then the system is referred to as **IU-ON** process, that is the typical case in digital to analog conversion path (timing-mismatch only @ output signal holding), e.g. TI DACs [2] or multirate sampled-data interpolators [5], as shown in Fig.1(b). If the **I**nput signal is sampled by the system with **N**onuniform time-interval then played out **C**orrelatively at **O**utput with the same **N**onuniform time instants from the input sampling, such system is named **IN-CON** process, that usually applies to a complete TI sampled-data system (timing-mismatches @ both input sampling and output holding driven by the same clock phases), as illustrated in Fig.1(c), e.g. N-path sampled-data filtering [3].

The signal spectra for these processes with Impulse-Sampled (IS) sequence form have been analyzed in [6] and [7], respectively. However, in practice, the real output signals are always in Sample-and-Hold (SH) or holding nature in the latter two processes, as shown waveforms in Fig.1. Fig.2 presents the plots of FFT output spectra of all IN-OU, IU-ON and IN-CON process with both IS and SH output for a sinusoidal input with normalized frequency $a=f_o/f_s=0.2$, timing-skew period M=8 and standard derivation of timing-skew ratio σ_{rm} = 0.1%. Unlike the case of IN-OU process, the output signal spectra of the latter two processes are not simply the $\sin(x)/x$ shaped version of the corresponding original impulse-sampled output due to the nonuniformly holding effect, e.g. compare the calculated signal-to-noise ratio and circled parts in the figure for all cases (the sideband magnitudes of SH version of the IN-OU output signal decrease gradually as the increase of the frequency due to the typical uniformly zero-order hold transfer function, while for the latter two cases there are irregular modifications for all the sidebands). Therefore, the previous analysis of IU-ON(IS) [7] and IN-CON(IS) [6] cannot be directly applicable for their corresponding SH versions. This paper will present a complete investigation of output signal spectra of practical IU-ON(SH) and IN-CON(SH) processes, including both the closed-form spectra expression as well as their output Signal-to-Noise-Ratio (SNR). The corresponding simulation results for illustrating the accuracy of the derived formula will then also be presented.

II. SIGNAL SPECTRA WITH NONUNIFORMLY HOLDING OUTPUT

In this section, the output spectra expression for both the IU-ON(SH) and IN-CON(SH) systems will be derived. Let us

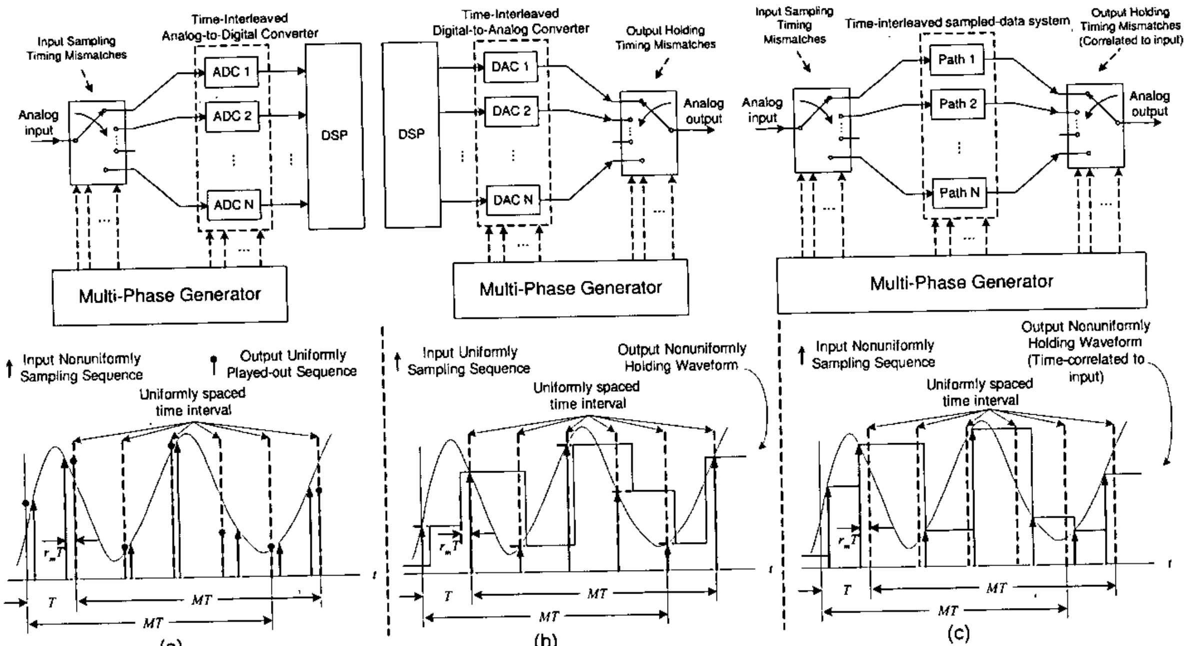


Fig. 1. Equivalent (a) IN-OU(IS) (b) IU-ON(SH) (c) IN-CON(SH) processes for Time-Interleaved ADC, DAC and Sampled-data Systems

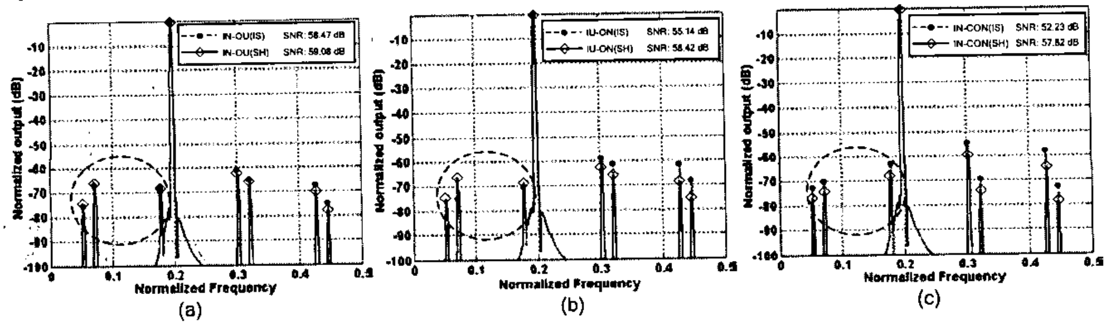


Fig. 2. FFT spectra of output sinusoid for (a) IN-OU, (b) IU-ON and (c) IN-CON processes

first consider the IU-ON(SH) system, which can be described by [6]

$$y(t)=\sum_{n=-\infty}^{\infty}x(nT)h_n(t-t_n) \tag{1}$$

with

$$t_n = nT+\Delta_n \tag{1a}$$

and $T=1/f_s$ is the nominal sampling period and Δ_n is a periodic skew timing sequence with period M (M is usually equal to TI path number N). For the nonuniformly holding output described in this paper, $h_n(t)$ can be expressed as:

$$h_n(t)=u(t)-u(t-T-\Delta_{n+1}+\Delta_n) \tag{2}$$

Let $n = kM+m$ ($m = 0,1,\ldots M$-1) and timing skew ratio $r_m=\Delta_m/T$, then we have

$$y(t)=\sum_{m=0}^{M-1}\sum_{k=-\infty}^{\infty}x(kMT+mT)h_m(t-kMT-mT-r_mT) \tag{3}$$

Applying the Fourier Transform to $y(t)$ and after simplification, the output spectrum of IU-ON(SH) system can be expressed as:

$$Y(\omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty}A_{2k\_SH}(\omega)\cdot X\left(\omega-k\frac{2\pi}{MT}\right) \tag{4}$$

where

$$A_{2k\_SH}(\omega)=\frac{1}{M}\sum_{m=0}^{M-1}H_m(\omega)e^{-jkm\frac{2\pi}{M}}e^{-j\omega r_mT} \tag{4a}$$

$$H_m(\omega)=\frac{2\sin(\omega(1+r_{m+1}-r_m)T/2)}{\omega}e^{-j\omega(1+r_{m+1}-r_m)T/2} \tag{4b}$$

and A_{2k_SH} is the weighted terms of modulation sidebands.

On the other hand, the output of IN-CON(SH) system can be described by

$$y(t)=\sum_{n=-\infty}^{\infty}x(t_n)h_n(t-t_n) \tag{5}$$

We can similarly obtain the weight terms to the modulated spectrum sideband of equation (4) as

$$A_{3k\_SH}(\omega)=\frac{1}{M}\sum_{m=0}^{M-1}H_m(\omega)e^{-jkr_m\frac{2\pi}{M}}e^{-jkm\frac{2\pi}{M}} \quad (6)$$

The equations (4), (4a), (4b) and (6) fully characterize the output signal spectra for IU-ON and IN-CON processes with output nonuniformly holding effects.

III. CLOSED-FORM EXPRESSION FOR SNR

To compute the SNR, we consider first the IU-ON(SH) system described in (4a). For a real input sinusoidal signal with frequency $\omega_o=2\pi f_o$, the sidebands are located at frequency of $\omega=\pm\omega_o+k(2\pi)/(MT)$. We equivalently evaluate the sideband components at only $\omega=\omega_o+k(2\pi)/(MT)$ over the range $[-f_s/2, f_s/2]$ to find the SNR over the range of $[0, f_s/2]$. In the following formula derivation, we assume $\omega_o\neq k(2\pi)/(MT)$, meaning that the signal (and also the sidebands) is not exactly located at integer multiple of f_s/M.

Simplifying (4a) by using $\omega=\omega_o+k(2\pi)/(MT)$ with also that r_m and $e^{-jkm\frac{2\pi}{M}}$ are periodic with period $m=M$, (4a) leads to

$$A_{2k\_SH}\left(\omega_0+k\frac{2\pi}{MT}\right)=\frac{2\sin(\omega_0T/2)}{\left(\omega_0+k\frac{2\pi}{MT}\right)M}e^{-j\omega_0T/2}\sum_{m=0}^{M-1}e^{-j\omega_0r_mT}e^{-jkr_m\frac{2\pi}{M}}e^{-jkm\frac{2\pi}{M}} \quad (7)$$

and the SNR can be found by the following formula [7]:

$$SNR_{2\_SH}=10\log_{10}\left[\frac{\left|A_{20\_SH}(\omega_o)\right|^2}{\sum_k\left|A_{2k\_SH}\left(\omega_o+k\frac{2\pi}{MT}\right)\right|^2}\right]dB \quad (8)$$

where the value of k in the summation is taken so that $-\pi f_s\leq\omega_o+k(2\pi)/(MT)\leq\pi f_s$. Assuming r_m, $m=0,1,2,\ldots M-1$ to be M independent, identically distributed (*i.i.d*) random variables with Gaussian distribution of zero mean and standard deviation of σ_{rm} ($=\sigma_t/T$ where σ_t is the standard deviation of timing jitter in seconds). Thus we can evaluate the expected values of the signal component $\left|A_{20\_SH}(\omega_o)\right|^2$ and the sideband components $\left|A_{2k\_SH}\left(\omega_o+k\frac{2\pi}{MT}\right)\right|^2$ from (7) as follows:

$$E\left[\left|A_{20\_SH}(\omega_0)\right|^2\right]=\frac{4}{\omega_0^2M^2}\sin^2(\omega_0T/2)\sum_{n=0}^{M-1}\sum_{m=0}^{M-1}E\left[e^{-j\omega_0(r_m-r_n)T}\right]\approx\frac{4}{\omega_0^2}\sin^2(\omega_0T/2) \quad (9)$$

$$E\left[\left|A_{2k\_SH}\left(\omega_0+k\cdot\frac{2\pi}{MT}\right)\right|^2\right]\approx\frac{4\sigma_{rm}^2T^2}{M}\sin^2(\omega_0T/2) \quad (10)$$

for small values of r_m such that $2\pi f_or_mT\ll1$. From (10) it is clear that the expected value of various noise components in IU-ON(SH) systems are identical for different values of k, thus the total noise power can be obtained by simply multiplying (10) by M-1. Finally using (9), (10) in (8), the SNR of IU-ON(SH) systems can be obtained as follows:

$$SNR_{2\_SH}=20\log_{10}\left(\frac{1}{2\pi a\sigma_{rm}}\right)-10\log_{10}\left(1-\frac{1}{M}\right) \quad (11)$$

where $a=f_o/f_s$ is the normalized frequency of the sinusoid.

Note that (11) describes also the pure random clock jitter by equivalently letting time-skew sequences period $M\to\infty$. Moreover, it is specifically that the formula (11) for IU-ON(SH) is identical to the SNR formula for IN-OU(IS) system [6], thus showing that the SNR for these two systems have the same expressions for small jitter errors. Indeed when $2\pi f_or_mT\ll1$, the normalized sideband patterns for IN-OU(IS) and IN-OU(SH) are similar (this is evident from the simulated spectra and SNR in Fig. 2).

For the IN-CON(SH) system, substituting $\omega=\omega_o+k(2\pi)/(MT)$ in (6) yields:

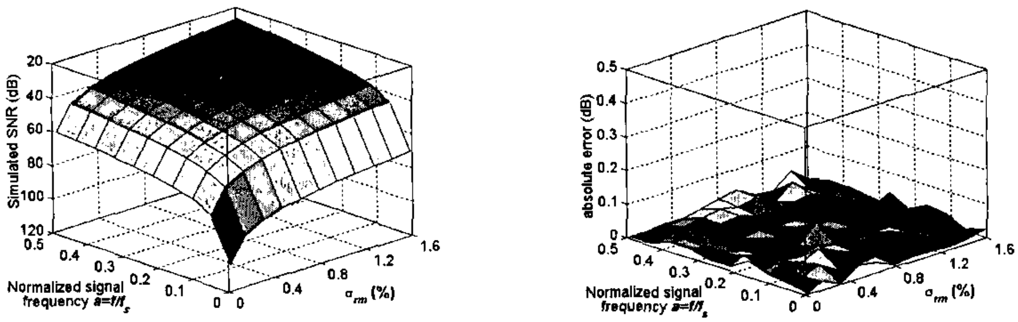
$$A_{3k\_SH}\left(\omega_0+k\frac{2\pi}{MT}\right)=\frac{1}{j\left(\omega_0+k\frac{2\pi}{MT}\right)M}\left\{\sum_{m=0}^{M-1}e^{-jkr_m\frac{2\pi}{M}}e^{-jkm\frac{2\pi}{M}}-e^{-j\omega_0T}\sum_{m=0}^{M-1}e^{-j\omega_0r_mT}e^{-jkr_m\frac{2\pi}{M}}e^{j\omega_0r_{m-1}T}e^{-jkm\frac{2\pi}{M}}\right\} \quad (12)$$

Using a similar approach, the expected value of signal component is just identical to (9), and the expected value of the noise components can be expressed as follows:

$$E\left[\left|A_{3k\_SH}\left(\omega_0+k\frac{2\pi}{MT}\right)\right|^2\right]\approx\frac{2\sigma_{rm}^2T^2}{M}\left\{2\sin^2(\omega_0T/2)-2\omega_0T\sin^2(\omega_0T/2)\cdot\frac{1-\cos(\omega_0T+k\frac{2\pi}{M})}{\omega_0T+k\frac{2\pi}{M}}+\omega_0^2T^2\cdot\frac{1-\cos(\omega_0T+k\frac{2\pi}{M})}{\left(\omega_0T+k\frac{2\pi}{M}\right)^2}-\omega_0T\sin(\omega_0T)\cdot\frac{\sin(\omega_0T+k\frac{2\pi}{M})}{\omega_0T+k\frac{2\pi}{M}}\right\} \quad (13)$$

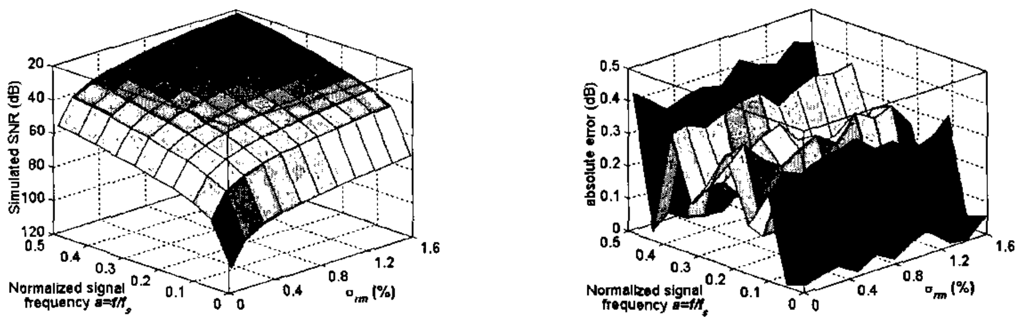
We can similarly derive the SNR for IN-CON(SH) systems with the exception that summation of (13) in (8) is approximated by integral due to the complexity of (13). Finally, we will arrive to the SNR as follows:

$$SNR_{3\_SH}\approx20\log_{10}\left(\frac{1}{2\pi a\sigma_{rm}}\right)-10\log_{10}\left[1-\frac{3.7a}{\tan(\pi a)}+\frac{7.64a^2}{\sin^2(\pi a)}\right] \quad (14)$$



(a) (b)

Fig. 3. (a) Simulated SNR & (b) absolute error between the simulated and calculated SNR for IU-ON(SH) process vs. normalized frequency a and standard derivation σ_{rm} by 10^4 times Monte Carlo Simulations (M=8)



(a) (b)

Fig. 4. (a) Simulated SNR & (b) absolute error between the simulated and calculated SNR of IN-CON(SH) process vs. normalized frequency a and standard derivation σ_{rm} by 10^4 times Monte Carlo Simulations (M=8)

Fig.3 and 4 shows the MATLAB simulation results (M=8) to illustrate the accuracy of the derived formula (11) & (14) compared with simulated SNR of IU-ON(SH) and IN-CON(SH) systems against the standard deviation σ_{rm} and normalized signal frequency $a=f_o/f_s$. For IU-ON(SH) system, the error between simulated SNR and that predicted by formula (11) is well below 0.1dB as shown in Fig. 3, and for IN-CON(SH) the error is within 0.5dB as shown in Fig. 4, thus demonstrating the effectiveness of the proposed theoretical prediction formulas.

IV. CONCLUSION

A complete and exact spectra analysis for nonuniformly holding signals has been presented in this paper which will be very useful to model accurately the timing-mismatch effects for practical time-interleaved systems. Due to the nonuniform nature of the sample-and-hold function in the time-domain, it has been shown that the output signal spectrum is not simply a $\sin(x)/x$ shaped version of the corresponding impulse-sampled signal spectrum. The closed-form expressions of both the signal spectra and SNR for IU-ON(SH) and IN-CON(SH) processes have been derived in this paper, and it also reveals the fact that the SNR for IN-OU(IS) is identical to that of IU-ON(SH) with the assumption of $2\pi f_o r_m T=2\pi a r_m$ <<1. Finally, the MATLAB simulations presented clearly demonstrate the effectiveness of the derived theoretical analysis results.

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